Critical field theory of the Kondo lattice model in two dimensions

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In the context of the U(1) slave boson theory we derive a critical field theory near the quantum critical point of the Kondo lattice model in two spatial dimensions. First we argue that strong gauge fluctuations in the U(1) slave boson theory give rise to confinement between spinons and holons, thus causing "neutralized" spinons in association with the slave boson U(1) gauge field. Second we show that critical fluctuations of Kondo singlets near the quantum critical point result in a new U(1) gauge field. This emergent gauge field has nothing to do with the slave boson U(1) gauge field. Third we find that the slave boson U(1) gauge field can be exactly integrated out in the low energy limit. As a result we find a critical field theory in terms of renormalized conduction electrons and neutralized spinons interacting via the new emergent U(1) gauge field. Based on this critical field theory we obtain the temperature dependence of specific heat and the imaginary part of the self-energy of the renormalized electrons. These quantities display non-Fermi liquid behavior near the quantum critical point.

It is now believed that the classical critical field theory of order parameter fluctuations, the Hertz-Millis theory of paramagnons in the Landau-Ginzburg-Wilson (*LGW*) theoretical framework, cannot explain the observed non-Fermi liquid behavior in thermal and electrical properties in the heavy fermion metals[1]. Although the *LGW* theoretical framework is the cornerstone of the theory of phase transitions, it is now necessary to go beyond the *LGW* theory. Recently Si et al. claimed that the non-Fermi liquid behavior can be explained by local quantum criticality[1]. According to their claim, electronic excitations resulting from the Kondo resonances participate in the critical theory, while in the traditional critical theory the fermionic excitations are just bystanders[1]. Another approach is based on an exotic phase of the U(1) slave boson theory, so called a deconfinement phase in the context of the gauge theory[2]. In the U(1) slave boson theory the critical theory is written in terms of not only order parameter fluctuations (holons) but also fermionic excitations (spinons). Furthermore, these spinons and holons interact via long range interactions mediated by the internal (slave boson) U(1) gauge fields. This critical theory successfully explains the non-Fermi liquid behavior in the Kondo problem[2].

In the present paper we reexamine the critical field theory in the U(1) slave boson representation. Despite the successful description of the non-Fermi liquid behavior[2] it is still controversial, especially in two dimensions. This is due to the fact that the U(1) gauge field is compact, thus admitting instanton excitations. In two dimensions the instanton excitations of the U(1) gauge field are believed to result in confinement[3] of the slave particles except some special cases[2, 3, 4, 5, 6, 7]. The slave boson critical theory does not belong to any special cases of the previous studies[2, 3, 4, 5, 6, 7]. If the confinement occurs, it is necessary to find a new critical field theory in terms of internal charge "neutral" particles emerging from the confinement. In the context of high T_c cuprates

it has been also pointed out that in the slave boson mean field theories strong gauge fluctuations mediating interactions between the slave particles are not appropriately taken into account[8, 9].

In this paper we derive a new critical field theory near the quantum critical point of the Kondo lattice model in two spatial dimensions. We find that the critical field theory can be written in terms of internal charge neutral fermions interacting via new emergent U(1) gauge fields [Eq. (11)]. The neutral fermions result from the confinement via the internal U(1) gauge field. The new emergent U(1) gauge field arises from critical fluctuations of Kondo singlets near the quantum critical point. Investigating the temperature dependence of specific heat and the imaginary part of the self-energy of the neutral fermions, we show that this new critical field theory can explain the anomalous behavior near the quantum critical point.

We consider the two dimensional Kondo lattice model in the U(1) slave boson representation

$$Z = \int Dc_\sigma Df_\sigma Da_\tau e^{-\int_0^\beta d\tau L},$$

$$L = \sum_k c_{k\sigma}^\dagger (\partial_\tau - \epsilon_k) c_{k\sigma} + \frac{J_K}{2} \sum_r \mathbf{S}_r \cdot c_{r\sigma}^\dagger \tau_{\sigma\sigma'} c_{r\sigma'}$$

$$+ \sum_r f_{r\sigma}^\dagger (\partial_\tau - ia_\tau) f_{r\sigma} + J_H \sum_{<r,r'>} \mathbf{S}_r \cdot \mathbf{S}_{r'}. \qquad (1)$$

Here c_σ is the conduction electron and f_σ, the spinon describing spin degrees of freedom of localized electrons. σ ranges over spin $\uparrow$ and $\downarrow$. $\mathbf{S}_r = \frac{1}{2} f_{r\sigma}^\dagger \tau_{\sigma\sigma'} f_{r\sigma'}$ is the localized spin. ϵ_k is the bare dispersion of the conduction electrons. a_τ is a Lagrange multiplier to guarantee a single occupancy constraint for spinons. It can be considered to be the time component of a gauge field as we will see later. J_K is the Kondo coupling constant between the conduction and localized spins, and J_H, the antiferromagnetic Heisenberg coupling constant between

localized spins.

Following Senthil et al.[2], we obtain a one-body effective Lagrangian in terms of the conduction electrons and spinons coupled to order parameters

$$Z = \int Dc_\sigma Df_\sigma DbDa_\mu e^{-\int_0^\beta d\tau L},$$

$$L = \sum_k c_{k\sigma}^\dagger(\partial_\tau - \epsilon_k)c_{k\sigma}$$

$$+ \sum_r f_{r\sigma}^\dagger(\partial_\tau - ia_\tau)f_{r\sigma} - \chi_0 \sum_{<r,r'>}(f_{r\sigma}^\dagger e^{ia_{rr'}}f_{r'\sigma} + h.c.)$$

$$- \sum_r(b_r^\dagger f_{r\sigma}c_{r\sigma}^\dagger + h.c.) + \sum_r \frac{4|b_r|^2}{J_K}. \quad (2)$$

Here b_r is the holon which represents hybridization between the conduction electron and spinon. $\chi_{rr'} = \chi_0 e^{ia_{rr'}}$ is the hopping order parameter of the spinons where $a_{rr'}$ is the internal U(1) gauge field originating from a composite field (slave boson) representation. The U(1) gauge field mediates long range interactions between the spinons and holons[2]. Mean field values of the order parameters are given by[2]

$$b_r^\dagger = \frac{J_K}{2} < c_{r\sigma}^\dagger f_{r\sigma} >,$$

$$\chi_0 = \frac{J_H}{2} < f_{r\sigma}^\dagger f_{r'\sigma} > . \quad (3)$$

Here we consider the case of half filling for the localized electrons as mentioned above, i.e., $1 = < f_{r\sigma}^\dagger f_{r\sigma} >$.

In passing, let us discuss mean field phases. In strong Kondo coupling regime the conduction electrons and spinons are strongly hybridized and thus the Kondo singlets are expected. This is represented by holon condensation, i.e., $< c_{r\sigma}^\dagger f_{r\sigma} > \neq 0$. Magnetic moments of the localized electrons are screened by conduction electrons and the local moments are expected to disappear. In this case not only the conduction electrons but also the spinons participate in conduction via the hybridization. Thus Fermi liquid with large Fermi surface is expected to occur[2]. In weak Kondo coupling regime the hybridization vanishes and the Kondo singlets disappear. The holons are not condensed, i.e., $< c_{r\sigma}^\dagger f_{r\sigma} > = 0$. The local magnetic moments are not screened and they are expected to appear via the antiferromagnetic coupling J_H. In this case the conduction electrons are decoupled from the spinons and only the conduction electrons participate in conduction[2]. Thus Fermi liquid with small Fermi surface is expected to appear[2].

At the critical Kondo coupling the hybridization is strongly fluctuating, causing critical fluctuations of the Kondo singlets. One may obtain a critical field theory for the order parameter (holon) fluctuations. New constituents in the critical field theory are fermionic excitations, here the spinons. The spinons interact with the holons via long range gauge interactions. Integrating over the conduction electrons, one can obtain the critical theory in terms of the spinons and holons interacting via the

U(1) gauge fields near the quantum critical point[2]. It is important to notice that the U(1) gauge field is compact. If the compactness is ignored, this critical theory successfully explains the non-Fermi liquid behavior in the heavy fermion metals[2]. However, as discussed earlier, it is generally accepted that owing to the instanton excitations originating from the compact U(1) gauge field only the confinement phase is expected to exist in two dimensions[3, 8]. Thus it is necessary to find a new critical field theory based on the confinement scenario. In addition, in the critical theory[2] the conduction electrons are considered to be bystanders but the spinons are not. Only the spinons and holons participate in the critical field theory. It seems that this is not appropriate to both conduction electrons and spinons. When we approach the quantum critical point from the strong Kondo coupling regime, critical fluctuations of the Kondo singlets are expected to affect both the conduction electrons and the spinons. Thus it seems to be natural that the critical theory should include not only the spinons but also the conduction electrons. Considering the above discussion, we can find natural conditions which the critical theory in the two dimensional Kondo problem should satisfy. The critical field theory should be written in terms of internal charge neutral particles as a result of the confinement. It has to include both the conduction electrons and spinons. Admitting the two conditions, we can construct the internal charge neutral particles in terms of the conduction electrons and spinons. Moreover, in order to explain the non-Fermi liquid behavior it is natural to consider long range gauge interactions between the internal charge neutral particles[10, 11, 12, 13, 14, 15]. We note that the gauge interaction here has nothing to do with the internal gauge field in the U(1) slave boson representation.

Our strategy is firstly to integrate over the holon field representing fluctuations of the Kondo singlets and to obtain an effective Lagrangian in terms of the conduction electrons and spinons. We approach the quantum critical point from the strong Kondo coupling phase, that is, the Kondo singlet paramagnetism. We will consider fluctuations of the phase of the holon field only, i.e., $b_r = b_0 e^{i\phi_r}$ with an amplitude $b_0 = \frac{J_K}{2}| < c_{r\sigma}^\dagger f_{r\sigma} > |$. The holon field carries both an internal charge and an electric charge associated with the internal U(1) gauge field a_μ and the electromagnetic field A_μ, respectively. The idea is to divide the phase field into partitions carrying the internal charge and the electric charge respectively. We rewrite the phase field as $\phi_r = \phi_{fr} + \phi_{cr}$. Here ϕ_{fr} carries only internal gauge charge and ϕ_{cr}, only electric charge. In order to treat the coupling $b_0 e^{-i\phi_r}f_{r\sigma}c_{r\sigma}^\dagger$ in Eq. (2) we consider the following gauge transformation

$$\tilde{f}_{r\sigma} = e^{-i\phi_{fr}}f_{r\sigma},$$

$$\tilde{c}_{r\sigma} = e^{i\phi_{cr}}c_{r\sigma}. \quad (4)$$

After performing the gauge transformation, we obtain $b_0 \tilde{f}_{r\sigma}\tilde{c}_{r\sigma}^\dagger$. Phase fluctuations of the holon field no longer couples to the conduction electrons and spinons in this

term. Instead the coupling appears in the kinetic term [Eq. (5)]. It is easy to treat this coupling as we discuss below. The phase field $\phi_{f(c)}(r)$ satisfies $\nabla \times \nabla \phi_{f(c)}(r) = 2\pi q_R \delta(r - R)$ where q_R is a vortex charge (integer) and R, the vortex center. This implies that in the case of non-zero q_R there exist points where $\phi_{f(c)}(r)$ is not well defined, i.e., singular. This transformation is usually called a singular gauge transformation[16]. The phase fields ϕ_{fr} and ϕ_{cr} are not pure gauge degrees of freedom, and thus vortex configurations ($q_R \neq 0$) should be allowed physically. As will be discussed below, the critical field theory is obtained by treating the vortex fields instead of the holon fields. In the present paper both the phase fields ϕ_{fr} and ϕ_{cr} will be taken into account. It should be noted that there is another singular gauge transformation of $\phi_{fr} = \phi_r$ and $\phi_{cr} = 0$. Utilizing this gauge transformation, we can also obtain a critical theory near the quantum critical point. In appendix A we show that this critical theory Eq. (A5) is totally different from our critical theory Eq. (11). We find that the critical theory Eq. (A5) cannot explain the non-Fermi liquid behavior in the Kondo system.

In the singular gauge transformation Eq. (4) we attach $e^{-i\phi_{fr}}$ to the spinon. As a result the internal gauge charge of the spinon is screened or neutralized by the phase. The renormalized spinon $\tilde{f}_{r\sigma}$ carries no internal gauge charge. This result can be interpreted as follows. As discussed earlier, strong gauge fluctuations a_μ are expected to cause the confinement of the internal gauge charges in $(2 + 1)D$[3, 7]. The gauge fluctuations confine the spinons and holons, resulting in the renormalized spinons which are neutral for the internal gauge charge. This point of view is analogous to that of Ref. [7, 17] in the context of high T_c cuprates. As will be seen below, the internal U(1) gauge field a_μ can be integrated out exactly in the low energy limit without affecting the neutralized spinons. The renormalized conduction electron $\tilde{c}_{r\sigma}$ is also neutralized. As we approach the quantum critical point, fluctuations of the hybridization between the conduction electrons and spinons are very strong. These fluctuations are expected to result in the renormalization of the conduction electrons. Although both the renormalized electrons and spinons are electrically neutral, it will be shown that both particles are coupled to the electromagnetic field A_μ in our critical field theory [Eq. (11)].

We rewrite the above Lagrangian [Eq. (2)] in terms of the renormalized fields [Eq. (4)]

$$L = \sum_r \tilde{c}_{r\sigma}^\dagger (\partial_\tau - i\partial_\tau \phi_{cr} - \epsilon(\sqrt{-(\nabla - i\nabla\phi_{cr})^2}))\tilde{c}_{r\sigma}$$
$$+ \sum_r \tilde{f}_{r\sigma}^\dagger (\partial_\tau + i\partial_\tau \phi_{fr} - ia_\tau)\tilde{f}_{r\sigma}$$
$$- \chi_0 \sum_{<r,r'>} (\tilde{f}_{r\sigma}^\dagger e^{-i(\nabla\phi_{fr} - a_{rr'})}\tilde{f}_{r'\sigma} + h.c.)$$
$$- \sum_r (b_0 \tilde{f}_{r\sigma}\tilde{c}_{r\sigma}^\dagger + h.c.) + \sum_r \frac{4b_0^2}{J_K}. \qquad (5)$$

Here we have rewritten the kinetic energy of the conduc-tion electrons in real space. As a result of the singular gauge transformation [Eq. (4)] phase fluctuations of the holon fields are now coupled to both the renormalized electrons and spinons in the kinetic energy terms. As will be seen below, quantum fluctuations of the Kondo singlets result in long range gauge interactions between the two particles near the quantum critical point. In other words, the phase fields ϕ_{cr} and ϕ_{fr} are reinterpreted as two kinds of U(1) gauge fields.

We introduce two kinds of U(1) gauge fields usually called the "Doppler" gauge field v_μ and the "Berry" gauge field a_μ^B in the context of the quantum disordered $d - wave$ superconductor for high T_c cuprates[16]

$$v_\mu = \frac{1}{2}(\partial_\mu \phi_{fr} + \partial_\mu \phi_{cr}),$$
$$a_\mu^B = \frac{1}{2}(\partial_\mu \phi_{fr} - \partial_\mu \phi_{cr}). \qquad (6)$$

In the context of the quantum disordered $d - wave$ superconductor the Doppler gauge field represents super-current fluctuations and causes the Doppler shift in a quasiparticle spectrum. The Berry gauge field represents vortex fluctuations and encodes the Aharnov-Bohm phase when the quasiparticles are turning around the vortices. These two gauge fields are introduced in order to treat strong phase fluctuations of the Cooper pair. Our Doppler and Berry gauge fields are not related with those in the superconductor. Instead fluctuations of the holon field can be easily treated by introducing the two gauge fields, as will be seen below.

Representing the above Lagrangian [Eq. (5)] in terms of the two gauge fields [Eq. (6)], we obtain the following Lagrangian in the continuum limit

$$Z = \int D\tilde{c}_\sigma D\tilde{f}_\sigma Dv_\mu Da_\mu^B Da_\mu e^{-\int d^3x \mathcal{L}},$$
$$\mathcal{L} = \tilde{c}_\sigma(\partial_\tau - iv_\tau + ia_\tau^B + iA_\tau)\tilde{c}_\sigma$$
$$+ \frac{1}{2m_c}|(\nabla - i\mathbf{v} + i\mathbf{a}^B + i\mathbf{A})\tilde{c}_\sigma|^2$$
$$+ \tilde{f}_\sigma(\partial_\tau + iv_\tau + ia_\tau^B - ia_\tau)\tilde{f}_\sigma$$
$$+ \frac{1}{2m_f}|(\nabla + i\mathbf{v} + i\mathbf{a}^B - i\mathbf{a})\tilde{f}_\sigma|^2$$
$$- (b_0 \tilde{f}_{r\sigma}\tilde{c}_{r\sigma}^\dagger + h.c.) + \frac{4b_0^2}{J_K}$$
$$+ \frac{\rho}{2}|\partial_\mu \phi - a_\mu - A_\mu|^2 - i\lambda_c(\partial \times v - \partial \times a^B - J_{cV})$$
$$- i\lambda_f(\partial \times v + \partial \times a^B - J_{fV}) \qquad (7)$$

with m_c, the mass of the conduction electron and $m_f \sim \chi_0^{-1}$, that of the spinon. Here the kinetic energy of the renormalized conduction electrons is explicitly written in the usual non-relativistic form with an electromagnetic field $A_\mu = (A_\tau, \mathbf{A})$. The low energy Lagrangian of the holon field is also explicitly shown with the constraints given by two Lagrange multipliers λ_c and λ_f. Here $J_{cV} = \partial \times \partial \phi_c$ and $J_{fV} = \partial \times \partial \phi_f$ are the vortex currents of

the holon partitions carrying the electric charge and the internal charge, respectively. These constraints are used in order to impose Eq. (6). Remember that the phase of the holon field is rewritten in the partitions, i.e., $\phi = \phi_c + \phi_f$. ρ is a stiffness parameter proportional to the condensation amplitude b_0^2.

Now we integrate over phase fluctuations of the holon field at the quantum critical point. Performing a standard duality transformation of the holon Lagrangian in Eq. (7), we obtain an effective vortex Lagrangian for each partition (ϕ_c and ϕ_f) of the holon field

$$
\begin{aligned}
\mathcal{L}_\phi^{dual} &= |(\partial_\mu - i\tilde{\lambda}_{c\mu})\Phi_c|^2 + m^2|\Phi_c|^2 + \frac{u}{2}|\Phi_c|^4 \\
&+ |(\partial_\mu - i\tilde{\lambda}_{f\mu})\Phi_f|^2 + m^2|\Phi_f|^2 + \frac{u}{2}|\Phi_f|^4 \\
&- i(\tilde{\lambda}_c + \tilde{\lambda}_f)_\mu(\partial \times v)_\mu + i(\tilde{\lambda}_c - \tilde{\lambda}_f)_\mu(\partial \times a^B)_\mu \\
&+ \frac{1}{2\rho}|\partial \times c|^2 + i(\partial \times c)_\mu(a_\mu + A_\mu - 2v_\mu).
\end{aligned} \tag{8}
$$

Here Φ_c and Φ_f are the vortex fields of ϕ_c and ϕ_f respectively, and c_μ, the vortex gauge field. $\tilde{\lambda}_{c\mu}$ and $\tilde{\lambda}_{f\mu}$ result from the shifted Lagrange multipliers $\tilde{\lambda}_{c\mu} = \lambda_{c\mu} + c_\mu$ and $\tilde{\lambda}_{f\mu} = \lambda_{f\mu} + c_\mu$, respectively. m is a vortex mass and u, a coupling strength of self-interaction. The vortex mass is given by $m^2 \sim J_K - J_K^c$ where J_K^c is the critical Kondo coupling. When $J_K > J_K^c$, the vortex vacuum is energetically favorable. This corresponds to the strong Kondo coupling phase where the holons are condensed. In the opposite case vortex condensation is expected to occur. This corresponds to the weak Kondo coupling phase where the holons are not condensed. The quantum critical point is obtained at $J_K = J_K^c$. At the quantum critical point the hybridization between the conduction electrons and spinons is strongly fluctuating. This is represented by critical phase fluctuations of the holon field. In the vortex Lagrangian Eq. (8) $J_K = J_K^c$ leads the vortex mass to be zero. As a result critical phase fluctuations of the holon field are represented by critical vortex fluctuations. The original vortex gauge field c_μ is decoupled from the vortex fields. Instead the two fields $\tilde{\lambda}_{c\mu}$ and $\tilde{\lambda}_{f\mu}$ act as the vortex gauge fields mediating interactions between the vortices. A similar and detailed derivation in the context of high T_c cuprates can be found in Ref. [17]. After integrating over the vortex gauge field c_μ, we obtain a mass term $\frac{\rho}{2}|a_\mu + A_\mu - 2v_\mu|^2$. The mass is a relevant parameter in the renormalization group sense. Thus, in the low energy limit we can set $v_\mu = \frac{1}{2}(a_\mu + A_\mu)$.

Inserting the Doppler gauge field $v_\mu = \frac{1}{2}(a_\mu + A_\mu)$ into Eq. (7) and Eq. (8), and integrating over the critical vortex fluctuations and the effective vortex gauge fields $\tilde{\lambda}_{c\mu}, \tilde{\lambda}_{f\mu}$ in Eq. (8) at the quantum critical point $J_K =$

J_K^c, we obtain an effective Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \tilde{c}_\sigma(\partial_\tau + ia_\tau^B - \frac{i}{2}a_\tau + \frac{i}{2}A_\tau)\tilde{c}_\sigma \\
&+ \frac{1}{2m_c}|(\nabla + i\mathbf{a}^B - \frac{i}{2}\mathbf{a} + \frac{i}{2}\mathbf{A})\tilde{c}_\sigma|^2 \\
&+ \tilde{f}_\sigma(\partial_\tau + ia_\tau^B - \frac{i}{2}a_\tau + \frac{i}{2}A_\tau)\tilde{f}_\sigma \\
&+ \frac{1}{2m_f}|(\nabla + i\mathbf{a}^B - \frac{i}{2}\mathbf{a} + \frac{i}{2}\mathbf{A})\tilde{f}_\sigma|^2 \\
&+ \frac{1}{2g_B^2}(\partial \times a^B)\frac{1}{\sqrt{-\partial^2}}(\partial \times a^B) \\
&+ \frac{1}{2g_A^2}(\partial \times a + \partial \times A)\frac{1}{\sqrt{-\partial^2}}(\partial \times a + \partial \times A) \\
&- (b_0\tilde{f}_{r\sigma}\tilde{c}_{r\sigma}^\dagger + h.c.) + \frac{4b_0^2}{J_K}.
\end{aligned} \tag{9}
$$

Here the effective couplings $g_B^2 = \left(\frac{8}{N_c} + \frac{8}{N_f}\right)^{-1}$ and $g_A^2 = \left(\frac{2}{N_c} + \frac{2}{N_f}\right)^{-1}$ are obtained in the usual $1/N$ expansion[7, 17] where $N_{c(f)}$ is the flavor number of the vortex $\Phi_{c(f)}$. In the present case we have $N_c = N_f = 1$. Anomalous kinetic terms for both the Berry gauge field and internal gauge field result from critical vortex fluctuations. Shifting the internal U(1) gauge field a_μ to $\tilde{a}_\mu = a_\mu + A_\mu$ and the Berry gauge field a_μ^B to $\tilde{a}_\mu^B = a_\mu^B - \frac{1}{2}\tilde{a}_\mu$ in Eq. (9), we obtain

$$
\begin{aligned}
\mathcal{L} &= \tilde{c}_\sigma(\partial_\tau + i\tilde{a}_\tau^B + iA_\tau)\tilde{c}_\sigma + \frac{1}{2m_c}|(\nabla + i\tilde{\mathbf{a}}^B + i\mathbf{A})\tilde{c}_\sigma|^2 \\
&+ \tilde{f}_\sigma(\partial_\tau + i\tilde{a}_\tau^B + iA_\tau)\tilde{f}_\sigma + \frac{1}{2m_f}|(\nabla + i\tilde{\mathbf{a}}^B + i\mathbf{A})\tilde{f}_\sigma|^2 \\
&+ \frac{1}{2g_B^2}(\partial \times \tilde{a}^B + \frac{1}{2}\partial \times \tilde{a})\frac{1}{\sqrt{-\partial^2}}(\partial \times \tilde{a}^B + \frac{1}{2}\partial \times \tilde{a}) \\
&+ \frac{1}{2g_A^2}(\partial \times \tilde{a})\frac{1}{\sqrt{-\partial^2}}(\partial \times \tilde{a}) \\
&- (b_0\tilde{f}_{r\sigma}\tilde{c}_{r\sigma}^\dagger + h.c.) + \frac{4b_0^2}{J_K}.
\end{aligned} \tag{10}
$$

Now the internal U(1) gauge field a_μ is decoupled from the renormalized spinon field. This result seems to be quite natural because the renormalized spinon is neutral under the internal gauge field. We can easily check whether the local gauge symmetry in the original Lagrangian Eq. (2) is preserved in all previous steps. The Lagrangian Eq. (2) has a $U_a(1) \times U_A(1)$ local gauge symmetry in association with the internal and electromagnetic U(1) gauge fields respectively. We note that the local gauge symmetry in Eq. (2) is preserved in all steps.

Integrating over the internal U(1) gauge field $\tilde{a}_\mu$ in Eq. (10), we finally obtain a critical field theory in terms of the renormalized conduction electrons and renormalized

spinons interacting via the effective U(1) gauge field $\tilde{a}_\mu^B$

$$\mathcal{L} = \tilde{c}_\sigma(\partial_\tau + i\tilde{a}_\tau^B + iA_\tau)\tilde{c}_\sigma + \frac{1}{2m_c}|(\nabla + i\tilde{\mathbf{a}}^B + i\mathbf{A})\tilde{c}_\sigma|^2$$

$$+\tilde{f}_\sigma(\partial_\tau + i\tilde{a}_\tau^B + iA_\tau)\tilde{f}_\sigma + \frac{1}{2m_f}|(\nabla + i\tilde{\mathbf{a}}^B + i\mathbf{A})\tilde{f}_\sigma|^2$$

$$+\frac{1}{2\tilde{g}^2}(\partial \times \tilde{a}^B)\frac{1}{\sqrt{-\partial^2}}(\partial \times \tilde{a}^B)$$

$$-(b_0\tilde{f}_{r\sigma}\tilde{c}_{r\sigma}^\dagger + h.c.) + \frac{4b_0^2}{J_K} \qquad (11)$$

with an effective coupling strength $\tilde{g}^2 = g_B^2 + \frac{1}{4}g_A^2$. Physically it is clear that the massless effective U(1) gauge field $\tilde{a}_\mu^B$ originates from critical fluctuations of the Kondo singlets (critical phase fluctuations of the holons) at the quantum critical point. Mathematically this new emergent gauge structure results from the composite structure in the phase of the holon field, i.e., $\phi_r = \phi_{fr} + \phi_{cr}$. The phase ϕ_r is invariant under the transformation $\phi_{fr} \to \phi_{fr} + \vartheta$ and $\phi_{cr} \to \phi_{cr} - \vartheta$. It is easy to verify that the critical field theory Eq. (11) has a new local gauge symmetry. Under the gauge transformation $\phi_{fr} \to \phi_{fr} + \vartheta$ and $\phi_{cr} \to \phi_{cr} - \vartheta$ the renormalized electrons and spinons are also transformed by $\tilde{c}_\sigma' = e^{-i\vartheta}\tilde{c}_\sigma$ and $\tilde{f}_\sigma' = e^{-i\vartheta}\tilde{f}_\sigma$, respectively. If the gauge field $\tilde{a}_\mu^B$ is transformed by $\tilde{a}_\mu'^B = \tilde{a}_\mu^B + \partial_\mu\vartheta$, this symmetry is preserved. This new gauge symmetry dictates a new U(1) gauge field $\tilde{a}_\mu^B$.

A critical field theory in the two dimensional Kondo lattice model is required to satisfy three conditions as discussed earlier. First the theory should consist of internal charge neutral particles as a result of confinement. Second it should be the case that not only the spinons but also the conduction electrons participate in the critical theory. Third the critical theory should explain the observed non-Fermi liquid behavior near the quantum critical point[1, 2]. A possible candidate is to introduce long range gauge interactions[10, 11, 12, 13, 14, 15] between the internal charge neutral particles. The critical field theory Eq. (11) satisfies the above three conditions. Eq. (11) is written in terms of both the conduction electrons and spinons. They interact via long range gauge interactions mediated by the U(1) gauge field $\tilde{a}_\mu^B$. Both particles are renormalized to be neutral under internal charge.

Now we discuss physics of the two dimensional Kondo lattice model based on the confinement scenario of the present paper. In the strong Kondo coupling regime of $J_K > J_K^c$ the conduction electrons and spinons are strongly hybridized. This is represented by the holon condensation as discussed earlier. In this case fluctuations of the Kondo singlets are suppressed. In the effective Lagrangian of the holon vortex field [Eq. (8)] $J_K > J_K^c$ leads to a positive vortex mass. As a result the condensation of the holons are represented by the vacuum of the vortices. In the vortex vacuum all gauge fields, a_μ, v_μ, and a_μ^B, become massive. This is consistent with the suppression of Kondo singlet fluctuations. Thus usual mean field treatment is available.

Fermi liquid behavior is expected owing to the suppression of gauge fluctuations[2]. In the context of the gauge theory this phase corresponds to the Higgs-confinement phase[3]. It is natural that only neutral particles for the internal gauge charge appear in an effective field theory. The renormalized spinons and conduction electrons are neutral under the internal gauge charge.

Approaching the quantum critical point from the paramagnetic phase, fluctuations of the hybridization become strong. These fluctuations are represented by phase fluctuations of the holon fields. In the dual vortex formulation [Eq. (8)] $J_K = J_K^c$ results in massless vortex excitations. Thus critical phase fluctuations of the holon field are represented by critical vortex fluctuations as discussed before. These cause both the internal U(1) gauge field a_μ and the Berry gauge field a_μ^B to be massless. As we have seen above, the internal U(1) gauge field can be integrated out exactly in the low energy limit without affecting the two renormalized particles. Only one effective U(1) gauge field remains in our critical field theory. It is well known that when the kinetic energy of the gauge field $\tilde{a}_\mu^B$ is Maxwellian, the critical field theory Eq. (11) shows non-Fermi liquid behavior owing to scattering with the massless gauge fluctuations[2, 10, 11, 12, 13, 14, 15]. Specific heat is shown to be proportional to $T\ln T$ in $3D$ instead of the standard γT behavior[2, 13, 14] where T is temperature. In $2D$ it is proportional to $T^{2/3}$[2, 10], which is observed in $YbRh_2Si_2$ in the low temperature regime near the quantum critical point[2, 18]. Conductivity is shown to be proportional to $T^{-4/3}$ in $2D$ and $T^{-5/3}$ in $3D$[2, 11, 12]. These anomalous behaviors are not captured by usual Fermi liquid observed in the case when the gauge fluctuations are suppressed via the Anderson-Higgs mechanism in the strong Kondo coupling regime. However, in the present case the kinetic energy of the gauge field is not Maxwellian owing to the critical vortex fluctuations. As far as we know, this effective field theory Eq. (11) has not been examined before in the literature. Owing to the non-Maxwellian contribution of the gauge field $\tilde{a}_\mu^B$ temperature dependence in the specific heat and conductivity is expected to change. Here we consider the specific heat and the self-energy of the renormalized particles in two dimensions.

Integrating over the renormalized spinon and electron fields in Eq. (11), we obtain an effective action in terms of the effective gauge field $\tilde{a}_\mu^B$. Integration over the gauge field in the usual random phase approximation results in the following expression of free energy

$$\frac{F}{V} = -\int\frac{d^D q}{(2\pi)^D}\int\frac{d\omega}{2\pi}\coth\Big[\frac{\omega}{2T}\Big]\tan^{-1}\Big[\frac{ImD^T(q,\omega)}{ReD^T(q,\omega)}\Big] \qquad (12)$$

Here $D^T(q,\omega)$ is the transverse retarded propagator of the gauge field $\tilde{a}_\mu^B$ which is renormalized by the polarization of the spinons and electrons. V is volume of the system. This expression is valid for any theory where we sum a sequence of bubble-type diagrams[10]. The re-

tarded gauge field propagator is given by

$$D^T(q,\omega) = [\sqrt{-\omega^2 + q^2} - i\omega\sigma(q) + \chi_d q^2]^{-1}. \quad (13)$$

Here $\sigma(q)$ is the conductivity resulting from both the spinons and electrons. It is given by $\sigma(q) = \sigma_f(q) + \sigma_c(q) = (k_{0f} + k_{0c})q^{-1}$ in the clean limit[11]. $k_{0f(c)}$ is near the Fermi momentum which is about the inverse of the lattice spacing[11]. The term $i\omega\sigma(q)$ describes dissipation of the gauge field due to quasiparticle excitations. $\chi_d = \chi_f + \chi_c = (24\pi)^{-1}(m_f^{-1} + m_c^{-1})$ is the diamagnetic susceptibility for a free fermion in two dimensions[11]. Here $m_{f(c)}$ is the mass of the spinon (electron). The non-Maxwell contribution $\sqrt{-\omega^2 + q^2}$ appears in the kernel $D^T(q,\omega)$. Ignoring the $\chi_d q^2$ contribution in the kernel owing to the non-Maxwell contribution $\sqrt{-\omega^2 + q^2}$, we obtain the imaginary and real part of the kernel

$$
\begin{aligned}
ImD^T(q,\omega) &= \frac{k_0\omega q}{\omega^2(-q^2 + k_0^2) + q^4}, \\
ReD^T(q,\omega) &= \frac{q^2\sqrt{-\omega^2 + q^2}}{\omega^2(-q^2 + k_0^2) + q^4}
\end{aligned}
\quad (14)
$$

with $k_0 \equiv k_{0f} + k_{0c}$. Inserting these expressions into Eq. (12), we obtain the following expression for the free energy

$$\frac{F}{V} = -\int \frac{d^D q}{(2\pi)^D} \int d\omega \coth\left[\frac{\omega}{2T}\right] \tan^{-1}\left[\frac{k_0\omega}{q\sqrt{-\omega^2 + q^2}}\right] \quad (15)$$

In appendix B we perform the momentum and frequency integrals explicitly. As a result we find

$$\frac{F}{V} = \frac{F_0}{V} + \frac{k_0}{16\pi^2}\left(\frac{3}{2} - \ln k_0\right)T^2 - \frac{k_0}{16\pi^2}T^2\ln T, \quad (16)$$

where F_0/V is large negative depending on energy cutoff. The specific heat is then obtained to be in two dimensions

$$
\begin{aligned}
C_V &= -\frac{T}{V}\left(\frac{\partial^2 F}{\partial T^2}\right)_V \\
&= \frac{k_0 \ln k_0}{8\pi^2}T + \frac{k_0}{8\pi^2}T\ln T. \quad (17)
\end{aligned}
$$

In the above expression the logarithmic correction seems to be natural. In the free energy expression Eq. (15) one can find $q \times \tan^{-1}[k_0\omega/q^2]$ in two dimensions in the limit of $\omega \ll q$ [appendix B]. When dynamics of the gauge field is described by the usual Maxwell kinetic energy, it is given by $q \times \tan^{-1}[k_0\omega/q^3]$[11]. In three dimensions it is given by $q^2 \times \tan^{-1}[k_0\omega/q^3]$[10]. In Ref. [10] $q^2 \times \tan^{-1}[k_0\omega/q^3]$ is approximated to $\sim \omega/q$ in the frequency range of $(k_0\omega)^{1/3} < q$. In the region of $k_0\omega/q^2 < 1$ we can also approximate $q \times \tan^{-1}[k_0\omega/q^2]$ to $\sim \omega/q$ [appendix B]. Both the expressions have the same momentum and frequency dependence. Thus we can see that the unusual dynamics described by the non-Maxwell kinetic energy results in "three dimensional effect". As a consequence we obtain a logarithmic correction in the temperature dependence of the specific heat.

Next we consider the imaginary part in the self-energy of the renormalized conduction electron. In the present formulation the single electron propagator $G_{\sigma\sigma}(r,\tau) = < T_\tau[c_\sigma(r,\tau)c_\sigma^\dagger(0,0)] >$ can be represented in terms of the renormalized conduction electrons

$$
\begin{aligned}
G_{\sigma\sigma}(r,\tau) &= < T_\tau[c_\sigma(r,\tau)c_\sigma^\dagger(0,0)] > \\
&= < T_\tau[\tilde{c}_\sigma(r,\tau)e^{i\int_{(0,0)}^{(r,\tau)} dx_\mu \partial_\mu \phi_c(x_\mu)}\tilde{c}_\sigma^\dagger(0,0)] > \\
&= < T_\tau[\tilde{c}_\sigma(r,\tau)e^{-i\int_{(0,0)}^{(r,\tau)} dx_\mu \tilde{a}_\mu^B(x)}\tilde{c}_\sigma^\dagger(0,0)] > \quad (18)
\end{aligned}
$$

Here we will not calculate this gauge invariant green's function Eq. (18). Instead we investigate a simpler one $G_{\sigma\sigma}(r,\tau) = < T_\tau[\tilde{c}_\sigma(r,\tau)\tilde{c}_\sigma^\dagger(0,0)] >$. Our objective is to see how the non-Maxwell dynamics of the gauge field can modify the results in the case of the Maxwell dynamics. More extensive study of Eq. (18) will be performed in the near future. Following Ref. [11], we write down the expression for the imaginary part of the self-energy

$$
\begin{aligned}
\Sigma_c''(k,\epsilon_k) &= \int_0^\infty d\omega \int \frac{d^D k'}{(2\pi)^D}[n(\omega) + 1][1 - f(\epsilon_{k'})] \\
&\quad \times (k + k')_\alpha (k + k')_\beta (2m_c)^{-2} \\
&\quad \times (\delta_{\alpha\beta} - q_\alpha q_\beta/q^2)ImD^T(q,\omega) \\
&\quad \times \delta(\epsilon_k - \epsilon_{k'} - \omega). \quad (19)
\end{aligned}
$$

Here $q_\alpha = (k' - k)_\alpha$ is the transfer momentum between the renormalized electrons and the gauge bosons. $n(\omega)$ and $f(\omega)$ are the boson and fermion occupation numbers, respectively. We perform the momentum and frequency integration explicitly in appendix C. As a consequence we find

$$
\begin{aligned}
\Sigma_c''(k,\epsilon_k) &= \frac{k_c N_c(0)}{2\pi m_c^2} \int_0^{\xi_k} d\omega \int_0^\infty dq \frac{k_0\omega q}{\omega^2(-q^2 + k_0^2) + q^4} \\
&\approx \frac{k_c N_c(0)}{8\pi m_c^2}\xi_k \quad (20)
\end{aligned}
$$

with $\xi_k = \epsilon_k - \mu_c$. Here ϵ_k and μ_c are the bare dispersion and chemical potential of the electrons respectively. k_c and $N_c(0)$ are the Fermi momentum and density of states of the electrons respectively. The last expression $\Sigma_c'' \sim \xi_k$ is obtained in the limit of $\xi_k \ll k_0$ [see appendix C]. In the case of the Maxwell dynamics the self-energy is found to be $\Sigma'' \sim \xi_k^{2/3}$ in two dimensions[11] and $\Sigma'' \sim \xi_k$ in three dimensions[14]. Thus we find that the non-Maxwell dynamics of the gauge field also causes the three dimensional effect to the self-energy like the case of the specific heat. It is well known that Fermi liquid shows $\Sigma'' \sim \xi_k^2$. Our critical field theory Eq. (11) describes non-Fermi liquid near the quantum critical point.

In the weak Kondo coupling regime of $J_K < J_K^c$ the hybridization is expected to disappear. The condensation amplitude of the holon field is expected to vanish, i.e., $b_0 = 0$. In this case it is not clear how to apply our formulation to this regime. This is because the singular gauge transformation [Eq. (4)] may be meaningless owing to the vanishing amplitude. If the condensation am-

plitude remained finite like the case of the quantum disordered superconductor[16], our theory might have been meaningful. Especially, our theory could be applied to the weak Kondo coupling regime not far from the quantum critical point. In this case the holon vortices are condensed and the two gauge fields, a_μ and a_μ^B remain massless. One difference from the critical field theory is that the kinetic energy of the gauge fields is Maxwellian. An effective coupling strength between the renormalized particles and the Berry gauge field a_μ^B is given by the condensation amplitude of the holon vortex fields[16, 17].

To summarize, quantum fluctuations of the Kondo singlets result in renormalization of both the conduction electrons and spinons. Furthermore, these cause long range gauge interactions between the renormalized particles near the quantum critical point. As a consequence the critical field theory is found to consist of the renormalized electrons and spinons interacting via the new emergent U(1) gauge field. Investigating the specific heat and the self-energy of the renormalized electrons, we have found the non-Fermi liquid behavior near the quantum critical point.

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APPENDIX A

In appendix A we discuss a different singular gauge transformation. We show that this gauge transformation does not result in the non-Fermi liquid behavior near the quantum critical point. We rewrite the singular gauge transformation Eq. (4)

$$\tilde{f}_{r\sigma} = e^{-i\phi_{fr}} f_{r\sigma},$$
$$\tilde{c}_{r\sigma} = e^{i\phi_{cr}} c_{r\sigma}. \qquad (A1)$$

If we choose $\phi_{cr} = 0$ and $\phi_{fr} = \phi_r$ with the phase field ϕ_r of the holon ($b_r = b_0 e^{i\phi_r}$), we obtain $\tilde{c}_{r\sigma} = c_{r\sigma}$ and $\tilde{f}_{r\sigma} = e^{-i\phi_r} f_{r\sigma}$. In this gauge transformation the electron is the same as before. Only the spinon is renormalized to carry an electric charge but no internal gauge charge. Based on this gauge transformation we obtain an effective Lagrangian

$$\mathcal{L} = c_\sigma(\partial_\tau + iA_\tau)c_\sigma + \frac{1}{2m_c}|(\nabla + i\mathbf{A})c_\sigma|^2$$
$$+\tilde{f}_\sigma(\partial_\tau + i\partial_\tau\phi - ia_\tau)\tilde{f}_\sigma + \frac{1}{2m_f}|(\nabla + i\nabla\phi - i\mathbf{a})\tilde{f}_\sigma|^2$$
$$-(b_0\tilde{f}_{r\sigma}\tilde{c}_{r\sigma}^\dagger + h.c.) + \frac{4b_0^2}{J_K} + \frac{\rho}{2}|\partial_\mu\phi - a_\mu - A_\mu|^2. \quad (A2)$$

Redefining the internal gauge field a_μ as $a_\mu = \tilde{a}_\mu + \partial_\mu\phi_r$ (called the unitary gauge), we obtain the massive internal gauge field $\tilde{a}_\mu$ in the Kondo singlet paramagnetic phase where the phase of the holon field is coherent ($< e^{i\phi_r} > \neq 0$). Thus usual mean field treatment is applicable in this regime. This is the same result to the

effective field theory based on the singular gauge transformation Eq. (4). See the discussion below Eq. (11). In the present paper the Kondo singlet paramagnetic phase is not of our interest. Instead we concentrate on the region near the quantum critical point of the Kondo lattice model. In this case it is difficult to apply this unitary gauge owing to strongly fluctuating holon vortices [see the discussion of the unitary gauge in the paper of H. Kleinert and F. S. Nogueira, Nucl. phys. B **651**, 361 (2003)]. In order to treat this problem we introduce the Doppler and Berry gauge fields as shown in Eq. (6). In the gauge transformation of $\phi_{cr} = 0$ and $\phi_{fr} = \phi_r$ in Eq. (A1) the Doppler and Berry gauge fields are the same $v_\mu = a_\mu^B = \frac{1}{2}\partial_\mu\phi_r$.

We rewrite Eq. (A2) in terms of these two gauge fields with constraints

$$\mathcal{L} = c_\sigma(\partial_\tau + iA_\tau)c_\sigma + \frac{1}{2m_c}|(\nabla + i\mathbf{A})c_\sigma|^2$$
$$+\tilde{f}_\sigma(\partial_\tau + iv_\tau + ia_\tau^B - ia_\tau)\tilde{f}_\sigma$$
$$+\frac{1}{2m_f}|(\nabla + i\mathbf{v} + i\mathbf{a}^B - i\mathbf{a})\tilde{f}_\sigma|^2$$
$$-(b_0\tilde{f}_{r\sigma}\tilde{c}_{r\sigma}^\dagger + h.c.) + \frac{4b_0^2}{J_K}$$
$$+\frac{\rho}{2}|\partial_\mu\phi - a_\mu - A_\mu|^2 - i\lambda_c(\partial \times v - \partial \times a^B)$$
$$-i\lambda_f(\partial \times v + \partial \times a^B - J_V), \qquad (A3)$$

where $J_V = \partial \times \partial\phi_r$ is a holon vortex three current. Integration over the Lagrange multipliers $\lambda_{f\mu}$ and $\lambda_{c\mu}$ in Eq. (A3) recovers Eq. (A2). Now we integrate over the phase field ϕ_r of the holon near the quantum critical point. The methodology is the same as that in the text.

Performing the standard duality transformation of the holon Lagrangian in Eq. (A3), we obtain an effective Lagrangian for the holon vortex field

$$\mathcal{L}_\Phi = |(\partial_\mu - i\tilde{\lambda}_{f\mu})\Phi|^2 + m^2|\Phi|^2 + \frac{u}{2}|\Phi|^4$$
$$+\frac{1}{2\rho}|\partial \times c|^2 - i(\partial \times c)_\mu(a_\mu + A_\mu)$$
$$-i\lambda_c(\partial \times v - \partial \times a^B) - i(\tilde{\lambda}_{f\mu} - c_\mu)(\partial \times v + \partial \times a^B) \quad (A4)$$

Here Φ is the holon vortex field and c_μ, the vortex gauge field. $\tilde{\lambda}_{f\mu}$ results from the shifted Lagrange multiplier $\tilde{\lambda}_{f\mu} = \lambda_{f\mu} + c_\mu$. Integration over the vortex gauge field c_μ results in a mass term for the gauge fields, $\frac{\rho}{2}|v_\mu + a_\mu^B - a_\mu - A_\mu|^2$. Thus $v_\mu + a_\mu^B = a_\mu + A_\mu$ is obtained in the low energy limit. Integration over the Lagrange multiplier $\lambda_{c\mu}$ gives $v_\mu = a_\mu^B$. This is what we have expected in the gauge transformation of $\phi_{cr} = 0$ and $\phi_{fr} = \phi_r$ in Eq. (A1). As a result we obtain $a_\mu = 2a_\mu^B - A_\mu$.

Inserting $a_\mu = 2a_\mu^B - A_\mu$ and $v_\mu = a_\mu^B$ into Eq. (A3) and Eq. (A4), we obtain an effective Lagrangian in the

two dimensional Kondo lattice model

$$
\begin{aligned}
\mathcal{L} &= c_\sigma(\partial_\tau + iA_\tau)c_\sigma + \frac{1}{2m_c}|(\nabla + i\mathbf{A})c_\sigma|^2 \\
&+ \tilde{f}_\sigma(\partial_\tau + iA_\tau)\tilde{f}_\sigma + \frac{1}{2m_f}|(\nabla + i\mathbf{A})\tilde{f}_\sigma|^2 \\
&- (b_0\tilde{f}_{r\sigma}\tilde{c}^\dagger_{r\sigma} + h.c.) + \frac{4b_0^2}{J_K} \\
&+ |(\partial_\mu - i\tilde{\lambda}_{f\mu})\Phi|^2 + m^2|\Phi|^2 + \frac{u}{2}|\Phi|^4 \\
&- i2\tilde{\lambda}_{f\mu}(\partial \times a^B)_\mu.
\end{aligned} \tag{A5}
$$

Eq. (11) in the text can be reduced to this effective Lagrangian in the case of $\tilde{a}^B_\mu = 0$ obtained by the gauge transformation in appendix A. As shown in this effective Lagrangian, Kondo singlet fluctuations represented by holon vortex fluctuations are decoupled from the quasiparticles c_σ and $\tilde{f}_\sigma$. Thus mean field like behavior for the quasiparticles is expected to occur even near the quantum critical point. We know that this cannot explain the quantum critical behavior in the Kondo lattice model. This gauge transformation does not give us satisfactory results.

The problem associated with the singular gauge transformation was already discussed in the context of the quantum disordered $d - wave$ superconductor[16, 19]. The gauge transformation in appendix A is usually called the "Anderson gauge"[19] while that in the text, the "FT gauge"[16]. In the Anderson gauge the phase field of the Cooper pair living on links of the lattices is attached to only one kind of electrons with spin $\uparrow$ or spin $\downarrow$. Consider the Cooper pair term $|\Delta_0|e^{-i\phi_{rr'}}c_{\uparrow r}c_{\downarrow r'}$, where $|\Delta_0|$ and $\phi_{rr'}$ are the amplitude and phase of the Cooper pair respectively. In the long wave length limit of our interest we can perform the continuum approximation $a = |r - r'| \to 0$ with the lattice spacing a. This continuum approximation is well adopted in the low energy limit[16, 19]. Then we obtain the expression $|\Delta_0|e^{-i\phi_r}c_{\uparrow r}c_{\downarrow r}$ for the Cooper pair term. We consider the following singular gauge transformation

$$
\begin{aligned}
\tilde{c}_{\uparrow r} &= e^{-i\phi_{\uparrow r}}c_{\uparrow r}, \\
\tilde{c}_{\downarrow r} &= e^{-i\phi_{\downarrow r}}c_{\downarrow r}.
\end{aligned} \tag{A6}
$$

Here the phase fields $\phi_{\uparrow r}$ and $\phi_{\downarrow r}$ satisfy $\phi_r = \phi_{\uparrow r} + \phi_{\downarrow r}$. Inserting these expressions into the Cooper pair term, we obtain $|\Delta_0|\tilde{c}_{\uparrow r}\tilde{c}_{\downarrow r}$. The coupling between the phase of the Cooper pairs and the electrons now appears in the kinetic energy term of the electrons[16, 19]. In Eq. (A6) $\phi_{\uparrow r} = \phi_r$ and $\phi_{\downarrow r} = 0$ is the Anderson gauge[19]. On the other hand, in the FT gauge the phases are randomly chosen[16]. Basically the present Kondo problem seems to be similar to the problem of the quantum disordered superconductor. In the Anderson gauge dynamics of the Cooper pairs is also decoupled from that of the BCS quasiparticles[19] like the above effective Lagrangian Eq. (A5). It is difficult to determine which is better a priori. The theory that explains experiments well may be considered as the correct one. But we can argue that both

the spinons and conduction electrons have the same position. Thus Kondo singlet fluctuations are expected to cause the same effect to both the spinons and electrons. Our effective field theory Eq. (11) shows this symmetric property.

APPENDIX B

In appendix B we perform the momentum and frequency integrals in Eq. (15) to obtain Eq. (16). We concentrate on quantum fluctuations in Eq. (15), i.e., $\omega > T$, so that $coth\left[\frac{\omega}{2T}\right]$ may be replaced by unity[20]. We obtain the following expression in two dimensions

$$
\begin{aligned}
\frac{F}{V} &= -\int \frac{d^D q}{(2\pi)^D} \int_0^\infty \frac{d\omega}{2\pi} coth\left[\frac{\omega}{2T}\right] tan^{-1}\left[\frac{k_0\omega}{q\sqrt{-\omega^2 + q^2}}\right] \\
&= -\frac{1}{4\pi^2}\int_0^\infty dqq \int_0^\infty d\omega coth\left[\frac{\omega}{2T}\right] tan^{-1}\left[\frac{k_0\omega}{q\sqrt{-\omega^2 + q^2}}\right] \\
&\approx -\frac{1}{4\pi^2}\int_0^\infty dqq \int_T^{v_c q} d\omega tan^{-1}\left[\frac{k_0\omega}{q\sqrt{-\omega^2 + q^2}}\right].
\end{aligned} \tag{B1}
$$

Here v_c is the Fermi velocity of the conduction electron. We are interested in how this quantity depends on the lower cutoff T in the frequency integral. For small frequencies $\omega << v_c k_c$ with the electron Fermi momentum k_c, the q integration can be done first[20]. In addition, in this limit we can approximate $tan^{-1}\left[k_0\omega/q\sqrt{-\omega^2 + q^2}\right]$ to $tan^{-1}\left[k_0\omega/q^2\right]$. Introducing the energy cutoff $\omega_c < v_c k_c$, we find the free energy as a function of temperature

$$
\begin{aligned}
\frac{F}{V} &\approx -\frac{1}{4\pi^2}\int_T^{\omega_c} d\omega \int_0^\infty dqq \, tan^{-1}\left[\frac{k_0\omega}{q^2}\right] \\
&= -\frac{k_0}{8\pi^2}\int_T^{\omega_c} d\omega\left[(1 - lnk_0)\omega - \omega ln\omega\right] \\
&= -\frac{k_0}{16\pi^2}\left(\frac{3}{2} - ln(k_0\omega_c)\right)\omega_c^2 \\
&+ \frac{k_0}{16\pi^2}\left(\frac{3}{2} - lnk_0\right)T^2 - \frac{k_0}{16\pi^2}T^2 lnT.
\end{aligned} \tag{B2}
$$

This momentum and frequency integration was done by Mathematica 5.0.

The above integration can be performed analytically in an appropriate approximation. If we concentrate on the frequency range $k_0\omega/q^2 < 1$, the $tan^{-1}[k_0\omega/q^2]$ function can be approximated to $k_0\omega/q^2$. This approximation is also used in Ref. [10]. As a result we obtain the following

expression of the free energy

$$
\begin{aligned}
\frac{F}{V} \ &\approx -\frac{1}{4\pi^2}\int_T^{\omega_c} d\omega \int_0^\infty dq\, q\, tan^{-1}\Big[\frac{k_0\omega}{q^2}\Big] \\
&\approx -\frac{1}{4\pi^2}\int_T^{\omega_c} d\omega \int_{\sqrt{k_0\omega}}^{k_0} dq\, \frac{k_0\omega}{q} \\
&= -\frac{k_0}{8\pi^2}\int_T^{\omega_c} d\omega\Big[\omega ln k_0 - \omega ln\omega\Big] \\
&= -\frac{k_0}{16\pi^2}\Big(\frac{1}{2}+ln(k_0/\omega_c)\Big)\omega_c^2 \\
&\quad +\frac{k_0}{16\pi^2}\Big(\frac{1}{2}+ln k_0\Big)T^2 - \frac{k_0}{16\pi^2}T^2 ln T. \quad (B3)
\end{aligned}
$$

This expression is almost the same as Eq. (B2) except some numerical factors. The specific heat can be easily evaluated from the above free energy Eq. (B3)

$$
\begin{aligned}
C_V \ &= -T\Big(\frac{\partial^2 F}{\partial T^2}\Big)_V \\
&= \frac{k_0}{8\pi^2}\Big(\frac{1}{2}-ln k_0\Big)T + \frac{k_0}{8\pi^2}T ln T. \quad (B4)
\end{aligned}
$$

This expression also has the same functional form with Eq. (17).

APPENDIX C

In appendix C we calculate the imaginary part in the self-energy of the conduction electron in the same way of Ref. [11]. We rewrite Eq. (18)

$$
\begin{aligned}
\Sigma_c''(k,\epsilon_k) \ &= \int_0^\infty d\omega \int \frac{d^D k'}{(2\pi)^D}[n(\omega)+1][1-f(\epsilon_{k'})] \\
&\quad \times (k+k')_\alpha (k+k')_\beta (2m_c)^{-2} \\
&\quad \times (\delta_{\alpha\beta}-q_\alpha q_\beta/q^2) Im D^T(q,\omega) \\
&\quad \times \delta(\epsilon_k - \epsilon_{k'} - \omega). \quad (C1)
\end{aligned}
$$

As usual, only states k' near the Fermi surface contribute and it is convenient to introduce the variable $\xi_k = \epsilon_k - \mu_c$[11]

$$
\begin{aligned}
\Sigma_c''(k,\epsilon_k) &= \frac{N_c(0)}{2\pi m_c^2}\int_0^\infty d\omega \int d\xi' d\theta\, \delta(\xi_k - \xi' - \omega) \\
&\times [n(\omega)+1][1-f(\xi')]|\mathbf{k}\times\hat{\mathbf{q}}|^2 \frac{k_0\omega q}{\omega^2(-q^2+k_0^2)+q^4} \quad (C2)
\end{aligned}
$$

Here θ is the angle between k and k' and thus the transfer momentum $q = k' - k$ is given by $q = 2k_c sin(\theta/2)$ with the Fermi momentum k_c. $N_c(0) = m_c/2\pi$ is the density of states of the electrons. In the above the explicit form of the gauge field propagator Eq. (14) is used. In the present study we concentrate on the case of $T = 0K$. Then the above expression becomes[11]

$$
\Sigma_c''(k,\epsilon_k) = \frac{k_c N_c(0)}{2\pi m_c^2}\int_0^{\xi_k} d\omega \int_0^\infty dq \frac{k_0\omega q}{\omega^2(-q^2+k_0^2)+q^4} \quad (C3)
$$

Performing the momentum and frequency integration analytically, we obtain the following expression of the self-energy $\Sigma_c''(k,\epsilon_k)$ in the limit of $\xi_k << k_0$

$$
\begin{aligned}
\Sigma_c''(k,\epsilon_k) &= \frac{k_c N_c(0)k_0}{2\pi m_c^2}\int_0^{\xi_k} d\omega\Big[\frac{\pi}{2}\frac{1}{\sqrt{4k_0^2-\omega^2}} \\
&\quad +\frac{1}{\sqrt{4k_0^2-\omega^2}}tan^{-1}\Big(\frac{\omega}{\sqrt{4k_0^2-\omega^2}}\Big)\Big] \\
&= \frac{k_c N_c(0)k_0}{4\pi m_c^2}\Big[\pi sin^{-1}\Big(\frac{\xi_k}{2k_0}\Big)+\Big(sin^{-1}\Big(\frac{\xi_k}{2k_0}\Big)\Big)^2\Big] \\
&\approx \frac{k_c N_c(0)}{8\pi m_c^2}\xi_k. \quad (C4)
\end{aligned}
$$

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